AFCRL-63-674

63-4-5

Research in Magnetohydrodynamics and Its Astrophysical Applications

BY

MAX KROOK

Harvard College Observatory
Harvard University
Cambridge 38, Massachusetts

Contract No. AF19 (604)—4545
Project No. 8635
Task No. 86352

FINAL REPORT

JUNE 1963

Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

AFCRL-63-674

RESEARCH IN MAGNETOHYDRODYNAMICS AND ITS ASTROPHYSICAL APPLICATIONS

by

Max Krook

Harvard College Observatory
Harvard University
Cambridge 38, Massachusetts

Contract No. AF19(604)-4545

Project No. 8635

Task No. 86352

FINAL REPORT

June 1963

Prepared for

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

Requests for additional copies by Agencies of the Department of Defense, their contractors, and other Government agencies, should be directed to the:

DEFENSE DOCUMENTATION CENTER (DDC)
CAMERON STATION
ALEXANDRIA, VIRGINIA

Department of Defense contractors must be established for DDC services or have their "need to know" certified by the cognizant military agency of their project or contract. All other persons and organizations should apply to the:

UNITED STATES DEPARTMENT OF COMMERCE
OFFICE OF TECHNICAL SERVICES
WASHINGTON 25, D. C.

RESEARCH IN MAGNETOHYDRODYNAMICS AND ITS

ASTROPHYSICAL APPLICATIONS

by

Max Krook

Abstract

The long-range goal of this contract was the theoretical study of magnetohydrodynamic phenomena in stellar atmospheres. It was clear, however, that an appreciable expansion in our knowledge of fundamental magnetohydrodynamic behavior would have to precede the fruitful quantitative study of such phenomena in stellar atmospheres. Moreover, available methods of calculating the structure of stellar atmospheres, even in the absence of magnetic fields, were so cumbersome as to make such calculations with magnetic fields prohibitively complicated and tedious.

The immediate aims of the investigation were therefore to study fundamental magnetohydrodynamic behavior and to evolve more efficient methods for solving the atmospheric structure equations. Keeping in mind the long-range goals of the project, an effort was made to construct dynamical models for stellar atmospheres and, in particular, dynamical models for transient phenomena in such atmospheres.

Magnetohydrodynamics and Plasma Physics

J. D. Murray and G. S. S. Ludford analyzed the flow of an electrically conducting fluid past a magnetized sphere in two papers distributed together as Scientific Report No. 1.

Two papers by M. Krook were published in 1959 that were related to the contract work. The first, "Shock Fronts in Ionized Gases" (Annals of Physics, 6, 188, 1959), gives a moment method for approximate solution of the Fokker-Planck equations for a plane shock wave in a plasma. The second paper, "Continuum Equations in the Dynamics of Rarefied Gases" (Journal of Fluid Mechanics, 6, 523, 1959), presents a general class of methods for the reduction of problems formulated microscopically in terms of kinetic theory, into approximately equivalent continuum problems; in particular Mott-Smith's method is generalized.

R. M. May has used the statistical model for molecular interactions to make an exhaustive analysis of the propagation of small-amplitude waves in an infinite, uniform, and uniformly magnetized plasma. In particular, he investigated the character of the propagation in the whole region from "low density" (collision frequency very small compared to plasma frequency and gyrofrequency) to "high density" (collision frequency large compared to plasma frequency and gyrofrequency).

In many situations of interest, we are confronted with the problem of solving the non-linear kinetic equations. These equations cannot

be solved exactly and approximate methods have therefore to be used. Different approximation methods often lead to very different solutions to the same problem. No way has yet been found for estimating the errors in such approximate solutions of the non-linear Boltzmann equations. To gain some insight into the accuracy of various types of approximation procedure, the following program was embarked upon:

Using the approximate kinetic equations appropriate to a statistical model for molecular interactions, numerically exact solutions to a number of problems can be obtained by first reducing the problem to one involving a set of non-linear integral equations with integral constraint conditions. The same problems can also be solved approximately by various methods, using the same statistical model. The various approximate solutions are then compared with the numerically exact solutions. The problems selected for this comparison were (i) Couette flow with heat transfer which depends on three dimensionless parameters (Mach number, Knudsen number, and temperature ratio of the walls) and (ii) the plane stationary shock wave which depends on Mach number. The numerically exact solutions for a wide range of the dimensionless parameters were obtained by D. Anderson and discussed in detail in his thesis, "Numerical Experiments in Kinetic Theory" (Department of Engineering and Applied Physics, Harvard University, 1963). Various types of approximate solutions to these same problems and the comparison with Anderson's solutions have been obtained by H. K. Macomber (not supported by this contract), and will be presented in the near future in his thesis.

Miss Barbara Abraham investigated the propagation of small-amplitude plasma waves in an ionized, exponential atmosphere (in the presence of a gravitational field). This work is relevant to the propagation of waves in a planetary ionosphere and in the solar corona. In addition, Miss Abraham and May investigated the effect of particle correlations on the propagation of waves in a plasma. These investigations were reported in Miss Abraham's thesis, "I. Propagation of Waves in a Plasma of Variable Density; II. Effect of Particle Correlations on Plasma Waves" (Physics Department, Harvard University, 1962).

E. H. Avrett has investigated the motion of a charged particle in the equatorial plane of a dipole field (Scientific Report No. 3).

Due to insufficient funds, the work of May, Anderson, and Miss Abraham, supported in part by this contract, will not be distributed as Scientific Reports. This work will, however, be published with due acknowledgment to the contract.

Structure of Stellar Atmospheres

The development of a rapid and efficient method of calculating the structure of a stellar atmosphere of given effective temperature, surface gravity, and chemical composition, proceeded through a sequence of steps. The accent here was placed on the general case of non-gray atmospheres.

P. H. Stone and J. E. Gaustad studied certain moment methods for the case of an absorption coefficient which is a step-function of frequency. G. F. Carrier and E. H. Avrett pointed out the importance of boundary layers for the problem and applied boundary layer techniques to the solution (Scientific Report No. 2).

Stone studied the same problem by use of approximate integral equations (Scientific Report No. 4).

M. Krook used the Poincaré-Lighthill perturbation method to solve the general non-gray problem. The zero-order solution in this method is formally that for a non-gray atmosphere. This method has been used by O. Gingerich to determine the structure of A-star atmospheres. However, the method is not suitable for very large departures from grayness. A more powerful and general perturbation-iteration method was then developed by Krook and Avrett. This method is not based on a gray solution in zero order and has been found to provide a very rapid and efficient technique for calculating atmospheric structure (Scientific Reports Nos. 5 and 6).

Solar Transient Phenomena

M. Krook has proposed a model for active regions in the solar atmosphere which seems capable, at least qualitatively, of accounting in a unified way for a wide range of observed transient phenomena. The model involves an interaction of strong localized magnetic fields with the convective motions in the hydrogen convective zone. The central idea is that

energy outflow is partially blocked locally by the strong field, and that an additional energy flux has thus to be carried by the surrounding material. This additional flux enhances the chromosphere-corona building activity of the hydrogen convective zone in the neighborhood of the strong local field. Various types of transient phenomena then occur depending on the orientation of the field (radial as in a sunspot field, or parallel to the surface), and on the character of the fringing fields. Qualitatively, several observable features of this model seem to agree with observational results. Quantitative studies of the model are now in progress.

The following Scientific Reports have been issued under this contract:

Scientific Report No. 1, August 1960, GRD-TN-60-499

On the Flow of a Conducting Fluid Past a Magnetized Sphere, by G. S. S. Ludford and J. D. Murray.

In the steady flow of an incompressible, inviscid, conducting fluid past a magnetized sphere, the first-order effects of the magnetic field and the conductivity are studied. Paraboloidal wakes of vorticity and magnetic intensity are formed, the former being half the size of the latter. The vorticity, generated by the non-conservative electromagnetic force, is logarithmically infinite on the sphere. For the case of a dipole of moment M at the centre of a sphere of radius a, the drag coefficient is

$$C_D = \frac{144\mu'^2}{5(2\mu+\mu')^2}\beta R_M,$$

where μ and μ' are the permeabilities of the fluid and sphere, respectively, β is the ratio of the representative magnetic pressure $\mu M^2/2a^6$ to the free-stream dynamic pressure, and R_M is the magnetic Reynolds number.

Further Results on the Flow of a Conducting Sphere Past a Magnetized Sphere, by G. S. S. Ludford and J. D. Murray.

In the present paper we consider the general axially symmetric magnetic distribution in more detail. It appears that the singularity in the vorticity can only be absent when the undisturbed magnetic field vanishes at the front stagnation point. Explicit formulas for the drag are given in terms of certain coefficients determined by the distribution; the drag is the same for image distributions with respect to the plane $\theta = \pi/2$. As an example we compute the drag due to an off-center dipole and find it to be larger than that for a centered dipole of the same moment.

<u>Scientific Report No. 2, January 1962, AFCRL-62-213</u>

The Application of a Moment Method to the Solution of Non-Gray Radiative-Transfer Problems, by Peter H. Stone and John E. Gaustad.

A modified version of Krook's moment method for solving equations of radiative transfer is presented. The method is then tested with a simple step-function form for the absorption coefficient by obtaining a large number

of numerical solutions in the zero-order approximation. The solutions show that the method is useful for a wide range of non-gray models and that a discontinuity in the absorption coefficient has the effect of decreasing the surface temperature relative to a gray atmosphere with the same effective temperature.

A Non-Gray Radiative Transfer Problem, by G. F. Carrier and E. H. Avrett.

The absorption coefficient for a stellar atmosphere depends on frequency as well as depth. Often the frequency dependence is characterized by a large discontinuity at each of the series limits. In order to investigate the effect of such a discontinuity, we consider a model atmosphere with an absorption coefficient of the simple form

$$\kappa(x,\nu) = \begin{cases} \kappa(x), & 0 \leq \nu < \nu_0, \\ \epsilon^{-1}\kappa(x), & \nu > \nu_0, \end{cases}$$

where ϵ is constant and small compared with unity. Using the Eddington approximation, we obtain for the temperature distribution a highly nonlinear differential equation which is of boundary-layer type. This equation is solved by conventional methods of boundary-layer theory. The calculated results exhibit a greatly reduced temperature in the outer layer of the atmosphere (the boundary layer) and a greatly reduced amount of ultraviolet flux ($\nu > \nu_0$) emerging from the atmosphere. The method by which the boundary-layer equation is obtained and solved is applicable for absorption

coefficientsof greater complexity and for approximations of higher order than the Eddington approximation.

Scientific Report No. 3, April 1962, AFCRL-62-283

Particle Motion in the Equatorial Plane of a Dipole Magnetic Field, by E. H. Avrett.

An exact relation is derived which describes bound particle orbits in the equatorial plane of a dipole magnetic field. An exact expression is then obtained for the average angular velocity of the particle about the dipole axis. The corresponding drift velocity is compared with the usual first-order expression based on a constant local field gradient. It is shown that the first-order expression for the drift velocity can be considerably in error when the particle loops are not small compared with the mean distance from the dipole axis.

Scientific Report No. 4, May 1963, AFCRL-63-487

Approximate Integral Equations for the Temperature in Non-Gray Model Atmospheres, by Peter H. Stone.

Approximate integral equations for the temperature distribution in non-gray model atmospheres in radiative equilibrium are derived and compared with the equations derived by using standard approximations. Use of the approximate equations greatly facilitates the numerical solution for the temperature distribution. Iteration procedures, implemented by

an IBM 7090, are applied to two approximate forms of the temperature equation for the special case where the monochromatic absorption coefficient is a step function of frequency, with the step's size independent of depth. Solutions for 56 representative models of this type are attempted and the convergence properties of the equations determined. The results show the effect of absorption discontinuities on atmospheric temperatures and thus indicate those spectral features that may be expected to have an important influence on atmospheric temperatures.

Scientific Report No. 5, June, 1963, (AFCRL ~~number to come~~ 63-819)

A Perturbation Method for Non-Gray Stellar Atmospheres, by Max Krook.

Poincaré-Lighthill perturbation procedure is applied to determine the temperature distribution in a non-gray stellar atmosphere. The zero-order solution is formally that for a gray atmosphere. The higher-order solutions involve both corrections to the temperature and scale changes.

Scientific Report No. 6, June, 1963, (AFCRL ~~number to come~~ 63-829)

The Temperature Distribution in a Stellar Atmosphere, by E. H. Avrett and Max Krook.

We describe an iterative perturbation method for the determination of the temperature distribution in a plane-parallel pure absorption atmosphere. The method is unrestricted by the nature of the absorption coefficient and is rapidly convergent.

UNCLASSIFIED

AF Cambridge Research Laboratories, Bedford, Mass.
AFCRL-63-674
RESEARCH IN MAGNETOHYDRODYNAMCIS AND ITS ASTROPHYSICAL APPLICATIONS, by Max Krook
Final Report, June 1963, 10 pp.

Unclassified Report

The long-range goal of this contract was the theoretical study of magnetohydrodynamic phenomena in stellar atmospheres. It was clear, however, that an appreciable expansion in our knowledge of fundamental magnetohydrodynamic behavior would have to precede the fruitful quantitative study of such phenomena in stellar atmospheres. Moreover, available methods of calculating the structure of stellar atmospheres, even in the absence of magnetic fields, were so cumbersome as to make such calculations with magnetic fields prohibitively complicated and tedious.

The immediate aims of the investigation were therefore to study fundamental magnetohydrodynamic behavior and to evolve more efficient methods for solving the atmospheric structure equations. Keeping in mind the long-range goals of the project, an effort was made to construct dynamical models for stellar atmospheres and, in particular, dynamical models for transient phenomena in such atmospheres.

1. Astrophysics
2. Theory of Stellar Atmospheres
3. Magnetohydrodynamics
4. Plasma Physics

I. AFCRL Project 8635
Task 86352
II. Contract AF19(604)-4545
III. Harvard College Observatory
IV. Krook, M.
V. In DDC collection

UNCLASSIFIED

UNCLASSIFIED

AF Cambridge Research Laboratories, Bedford, Mass.
AFCRL-63-674
RESEARCH IN MAGNETOHYDRODYNAMCIS AND ITS ASTROPHYSICAL APPLICATIONS, by Max Krook
Final Report, June 1963, 10 pp.

Unclassified Report

The long-range goal of this contract was the theoretical study of magnetohydrodynamic phenomena in stellar atmospheres. It was clear, however, that an appreciable expansion in our knowledge of fundamental magnetohydrodynamic behavior would have to precede the fruitful quantitative study of such phenomena in stellar atmospheres. Moreover, available methods of calculating the structure of stellar atmospheres, even in the absence of magnetic fields, were so cumbersome as to make such calculations with magnetic fields prohibitively complicated and tedious.

The immediate aims of the investigation were therefore to study fundamental magnetohydrodynamic behavior and to evolve more efficient methods for solving the atmospheric structure equations. Keeping in mind the long-range goals of the project, an effort was made to construct dynamical models for stellar atmospheres and, in particular, dynamical models for transient phenomena in such atmospheres.

1. Astrophysics
2. Theory of Stellar Atmospheres
3. Magnetohydrodynamics
4. Plasma Physics

I. AFCRL Project 8635
Task 86352
II. Contract AF19(604)-4545
III. Harvard College Observatory
IV. Krook, M.
V. In DDC collection

UNCLASSIFIED

UNCLASSIFIED

AF Cambridge Research Laboratories, Bedford, Mass.
AFCRL-63-674
RESEARCH IN MAGNETOHYDRODYNAMCIS AND ITS ASTROPHYSICAL APPLICATIONS, by Max Krook
Final Report, June 1963, 10 pp.

Unclassified Report

The long-range goal of this contract was the theoretical study of magnetohydrodynamic phenomena in stellar atmospheres. It was clear, however, that an appreciable expansion in our knowledge of fundamental magnetohydrodynamic behavior would have to precede the fruitful quantitative study of such phenomena in stellar atmospheres. Moreover, available methods of calculating the structure of stellar atmospheres, even in the absence of magnetic fields, were so cumbersome as to make such calculations with magnetic fields prohibitively complicated and tedious.

The immediate aims of the investigation were therefore to study fundamental magnetohydrodynamic behavior and to evolve more efficient methods for solving the atmospheric structure equations. Keeping in mind the long-range goals of the project, an effort was made to construct dynamical models for stellar atmospheres and, in particular, dynamical models for transient phenomena in such atmospheres.

1. Astrophysics
2. Theory of Stellar Atmospheres
3. Magnetohydrodynamics
4. Plasma Physics

I. AFCRL Project 8635
Task 86352
II. Contract AF19(604)-4545
III. Harvard College Observatory
IV. Krook, M.
V. In DDC collection

UNCLASSIFIED

UNCLASSIFIED

AF Cambridge Research Laboratories, Bedford, Mass.
AFCRL-63-674
RESEARCH IN MAGNETOHYDRODYNAMCIS AND ITS ASTROPHYSICAL APPLICATIONS, by Max Krook
Final Report, June 1963, 10 pp.

Unclassified Report

The long-range goal of this contract was the theoretical study of magnetohydrodynamic phenomena in stellar atmospheres. It was clear, however, that an appreciable expansion in our knowledge of fundamental magnetohydrodynamic behavior would have to precede the fruitful quantitative study of such phenomena in stellar atmospheres. Moreover, available methods of calculating the structure of stellar atmospheres, even in the absence of magnetic fields, were so cumbersome as to make such calculations with magnetic fields prohibitively complicated and tedious.

The immediate aims of the investigation were therefore to study fundamental magnetohydrodynamic behavior and to evolve more efficient methods for solving the atmospheric structure equations. Keeping in mind the long-range goals of the project, an effort was made to construct dynamical models for stellar atmospheres and, in particular, dynamical models for transient phenomena in such atmospheres.

1. Astrophysics
2. Theory of Stellar Atmospheres
3. Magnetohydrodynamics
4. Plasma Physics

I. AFCRL Project 8635
Task 86352
II. Contract AF19(604)-4545
III. Harvard College Observatory
IV. Krook, M.
V. In DDC collection

UNCLASSIFIED